



MUST KNOW



HIGH SCHOOL

GEOMETRY

SECOND EDITION

Find out the important stuff in a flash

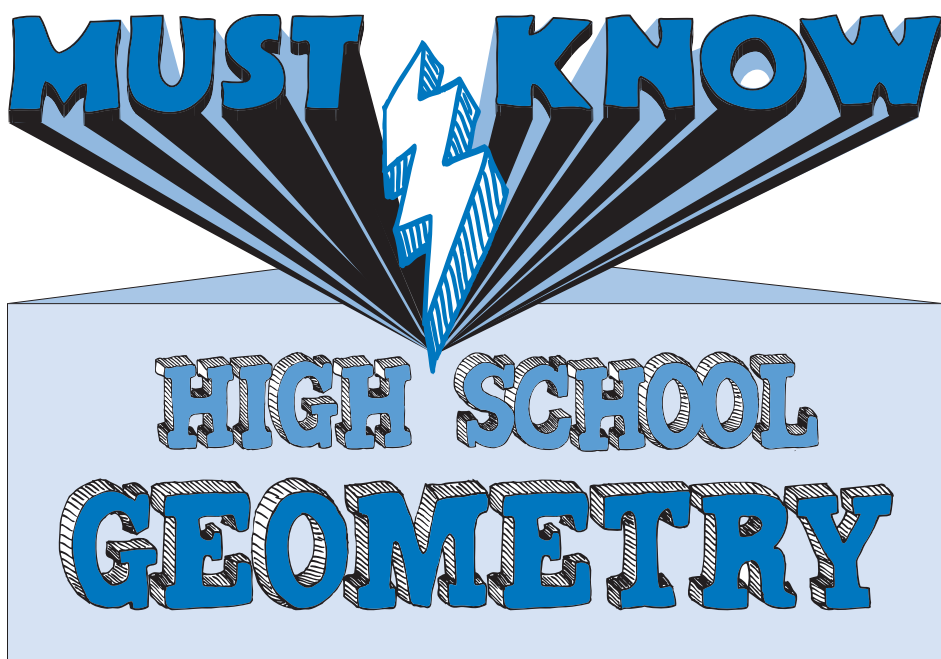
Solve geometry problems from the get-go

Allen Ma

Amber Kuang

**Mc
Graw
Hill**

**Bonus
Flashcard
App**

The title is presented in a dynamic, 3D style. The words "MUST KNOW" are at the top in a bold, blue, sans-serif font. A stylized lightning bolt, also in blue with black outlines, strikes down from the center of "MUST KNOW" towards the word "GEOMETRY". Below this, the words "HIGH SCHOOL" and "GEOMETRY" are stacked in a similar bold, blue, sans-serif font. The entire title is set against a light blue background that has a trapezoidal shape, wider at the top and narrower at the bottom, creating a sense of depth and focus.

MUST KNOW
HIGH SCHOOL
GEOMETRY

Second Edition

Allen Ma

Amber Kuang



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Dedication

We would like to dedicate this book to Lorraine Poppe, Madeline Donahoe, and Bill Ma. We are forever grateful for the endless encouragement and inspiration you have provided us. Thank you for believing in us before we believed in ourselves. You truly taught us the **must knows** of being a great educator.

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Introduction




Welcome to your new geometry book! Let us try to explain why we believe you've made the right choice with this new edition. This probably isn't your first go-round with either a textbook or other kind of guide to a school subject. You've probably had your fill of books asking you to memorize lots of terms. This book isn't going to do that—although you're welcome to memorize anything you take an interest in. You may also have found that a lot of books make a lot of promises about all the things you'll be able to accomplish by the time you reach the end of a given chapter. In the process, those books can make you feel as though you missed out on the building blocks that you actually need to master those goals.



With *Must Know High School Geometry*, we've taken a different approach. When you start a new chapter, right off the bat you will immediately see one or more **must know** ideas. These are the essential concepts behind what you are going to study, and they will form the foundation of what you will learn throughout the chapter. With these **must know** ideas, you will have what you need to hold it together as you study, and they will be your guide as you make your way through each chapter, learning how to build new knowledge and skills on them.

To build on this foundation, you will find easy-to-follow discussions of the topic at hand, and these are accompanied by comprehensive examples that show you how to apply what you're learning to solve typical geometry questions. Each chapter ends with review questions—more than 300 throughout the book—that are designed to instill confidence as you practice your new skills.

This book has other features that will help you on this geometry journey of yours. It has a number of sidebars that will provide helpful information or just serve as a quick break from your studies. The **BTW** sidebars (“by the



way”) point out important information as well as study tips and exceptions to the rule. Every once in a while, an  **IRL** sidebar (“in real life”) will tell you what you’re studying has to do with the real world; other IRLs may just be interesting factoids.

But that’s not all—this new edition has taken it a step further. We know our geometry students well and we want to make sure you’re getting the most out of this book. We added new  sidebars that point out common mistakes and things *not* to do. For those needing a little assistance, we have our  feature, where more challenging concepts, topics, or questions are given some more explanation. And finally, one special note for the teachers (because we didn’t forget about you!)—a **Teacher’s Guide** section at the back of the book is a place where you can go to find tips and strategies on teaching the material in the book, a behind-the-scenes look at what the authors were thinking when creating the material, and resources curated specifically to make your life easier!

In addition, this book is accompanied by a flashcard app that will give you the ability to test yourself at any time. The app includes more than 100 “flashcards” with a review question on one “side” and the answer on the other. You can either work through the flashcards by themselves or use them alongside the book. To find out where to get the app and how to use it, go to “The Flashcard App.”

Now that you’re ready to get started, let us introduce you to your guides throughout this book. Between them, Allen Ma and Amber Kuang teach algebra, geometry, trigonometry, precalculus, calculus, and discrete math. They have a clear idea about what you should get out of a geometry course and have developed strategies to help you get there. They also have seen the kinds of trouble that students can run into, and they are experienced hands at solving those difficulties. In this book, they apply that experience both to showing you the most effective way to learn a given concept as well as how to extricate yourself from traps you may have fallen into. They will be trustworthy guides as you expand your geometry knowledge and develop new skills.



Before we leave you to your authors' capable guidance, let us give you one piece of advice. Although we know that saying something "is the *worst*" is a cliché, if anything in geometry *is* the worst, it's formal geometric proofs. Let your new teachers introduce you to them and show you how to apply them confidently to your geometry work. Mastering geometric proofs will give you an invaluable advantage for the rest of your math career.

Good luck with your studies!

The Editors at McGraw Hill

The Flashcard App

This book features a bonus flashcard app. It will help you test yourself on what you've learned as you make your way through the book (or in and out). It includes 100-plus "flashcards," with information and terms on both the "fronts" and "backs" of the cards. You can use the app in two ways: (1) You can jump right into the app and start from any point that you want. Or (2) you can take advantage of the handy QR codes near the end of each chapter in the book; they will take you directly to the flashcards related to what you're studying at the moment.

To take advantage of this bonus feature, follow these easy steps:

Search for **McGraw Hill Must Know App** from either Google Play or the App Store.



Download the app to your smartphone or tablet.



Once you've got the app, you can use it in either of two ways.



Just open the app and you're ready to go.

Use your phone's QR Code reader to scan any of the book's QR codes.

You can start at the beginning, or select any of the chapters listed.

You'll be taken directly to the flashcards that match your chapter of choice.






Be ready to test your geometry knowledge!

1

Definitions

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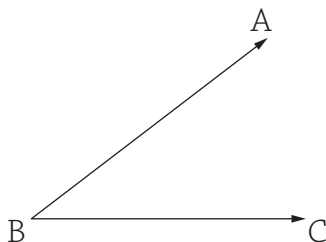
-  Terms and definitions in geometry provide the foundation for proving geometric theorems.
-  Geometric theorems enable us to develop the statements and reasons necessary to write a formal geometric proof.
-  The reflexive property tells us that an angle or segment is congruent to itself.



Mastering geometry requires the understanding of many properties and definitions. This chapter will focus on the different vocabulary that weaves its way through many geometry units. You might find it helpful to create index cards to help commit the information to memory. It is also critical to become familiar with the different symbols that are used to represent some of the vocabulary words.

The Basics

Let's begin by defining an **angle**. Represented by \angle , an angle is the space between two intersecting lines, line segments, or rays. Angles can be named using one letter if it is the only angle at that location, or three letters, the middle letter being the vertex of the angle. The accompanying diagram shows an angle that would be written as $\angle ABC$, where B is the vertex of the angle.

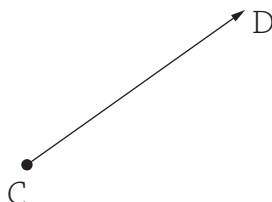


Another symbol we frequently see in geometry is the symbol for a **line**, \leftrightarrow . The symbol for a line has arrows on both sides to show that it extends infinitely in both directions. The accompanying diagram shows a line that would be written as \overleftrightarrow{AB} .





A **ray** begins at an endpoint and then continues infinitely in one direction. Therefore, a ray is represented by the symbol \rightarrow . The accompanying diagram shows a ray that would be written as \overrightarrow{CD} .



A **segment** has definite endpoints on both sides, which is why it is represented by the symbol $\overline{\quad}$. A segment does not have arrows indicating that it is a part of a line with definite endpoints on both sides. The accompanying diagram shows a segment that would be written as \overline{EF} .



Bisectors and Midpoints

Now that we are more familiar with some of the symbols that will appear throughout this book, we can begin learning some of the important geometric terms. Let's begin by learning about **bisectors**. There are two different types of bisectors: angle bisectors and segment bisectors.

A bisector divides an angle or segment into two congruent parts. This means that an **angle bisector** divides an angle into two congruent angles, and a **segment bisector** divides a segment into two congruent segments. In geometry, the symbol for congruence is \cong . When a segment is bisected, the point of intersection is called the **midpoint** of the segment. This means that the midpoint of a segment also divides a segment into two congruent segments.

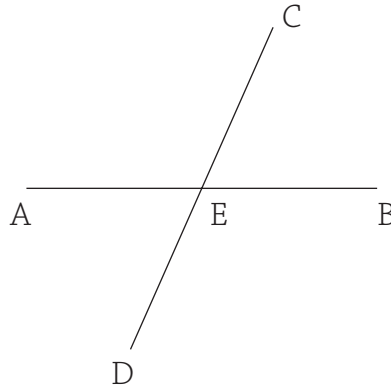
Let's look at some different problems that involve bisectors. Our first bisector example will help us understand how to develop information for a geometric proof.

BTW

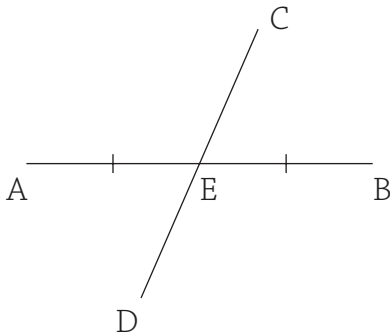
Read problems carefully. If a question says \overline{AB} is a bisector, that does not mean that \overline{AB} is being divided in half. It means \overline{AB} is dividing something else in half.


EXAMPLE

In the accompanying diagram, \overline{CD} bisects \overline{AB} at E . What can we conclude from this given information?



This question states that \overline{AB} is the segment being bisected. This means \overline{AB} will be divided into two congruent segments that make E the midpoint of \overline{AB} . Therefore, $\overline{AE} \cong \overline{EB}$. See the markings in the accompanying diagram that show $\overline{AE} \cong \overline{EB}$.



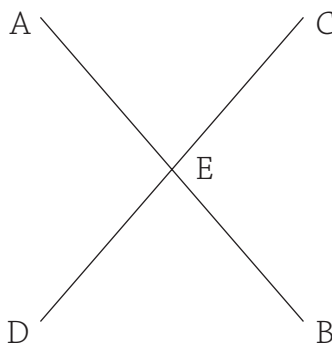
EASY MISTAKE
Do NOT conclude that \overline{CE} is congruent to \overline{DE} . E is NOT necessarily the midpoint of \overline{CD} .



Did you know it is possible for the bisector to be bisected? Let's look at an example.

EXAMPLE

▶ In the accompanying diagram, \overline{AB} and \overline{CD} bisect each other at E . What can we conclude from this given information?



▶ This question is stating that \overline{AB} and \overline{CD} are both being bisected, which means both segments will be divided into two congruent segments. Therefore, $\overline{AE} \cong \overline{EB}$ and $\overline{CE} \cong \overline{ED}$. See the markings in the accompanying diagram.

